

A New Model of Holographic Dark Energy with Action Principle

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ABSTRACT: We propose a new model of holographic dark energy with an action. It is the first time that one can derive a HDE model from the action principle. The puzzles of causality and circular logic about HDE have been completely solved in this model. The evolution of the universe only depends on the present state of the universe, clearly showing that it obeys the law of causality. Furthermore, the use of future event horizon as a present cut-off is not an input but automatically follows from equations of motion. Interestingly, this new model is very similar to the initial one of Li except a new term which may be explained as dark radiation.

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1 Introduction

The dark energy problem is a longstanding problem ever since the discovery of the accelerating expansion of the universe [1]. For a recent review of dark energy, please refer to [2]. Based on an idea of Cohen et al [3, 4], Li proposed the first model of the holographic dark energy (HDE) which can drive the accelerating expansion [5]. Though it is in good agreement with observational data [6], it causes some criticisms due to its use of the future event horizon as a present cut-off. These criticisms can be summarized as the causality problem and the circular logic problem. The causality problem: it seems that the evolution of the universe depends on the future information about the universe, the future event horizon. Beside, the equations of motion are non-local since the future event horizon is defined globally. The circular logic problem: the future event horizon exists only in an accelerating universe. How can one use an assumption based on the accelerating expansion to explain the accelerating expansion? For recent interesting discussions of these problems, please see [7]. For proposals using other infrared cut-off, see [8]-[10].

In this paper, we solve the causality problem and circular logic problem of HDE completely. First, we derive a new HDE model from the action principle. We find that we can rewrite all equations of motion (both the new and the initial HDE models) in local forms. Then the evolution of the universe only depends on the present initial conditions, which clearly obeys the law of causality. What is more, we do not need to assume the future event horizon as the infrared cut-off but only a local equation $\dot{L} = -\frac{1}{a}$. Magically, the equations of motion force the cut-off to be exactly the future event horizon aL . So it is not that the present cut-off depends on the future event horizon but conversely the future event horizon is completely determined by present cut-off through equations of motion.

This paper is arranged as follows. In Sec. 2, we derive a new HDE model from the action principle. We find exact solutions with matter and radiation and investigate cosmology in this model. In Sec. 3, we study models with other infrared cut-off in the action. We conclude in Sec. 4.

2 New HDE model from action principle

In this section, we derive a new model of holographic dark energy from the action principle. We find exact solutions of this model and prove that the cut-off is exactly the future event horizon. We also investigate cosmology of this model and find that radiation is dominant in early time, while dark energy is dominant in late time.

2.1 The general theory

Consider the Robertson-Walker metric

$$ds^2 = -N^2(t)dt^2 + a^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega^2\right] \quad (2.1)$$

and the following action

$$S = \frac{1}{16\pi} \int dt [\sqrt{-g}(R - \frac{2c}{a^2(t)L^2(t)}) - \lambda(t)(\dot{L}(t) + \frac{N(t)}{a(t)})] + S_M, \quad (2.2)$$

where R is the Ricci scalar, $\sqrt{-g} = Na^3$ (we have integrated the r, θ, ϕ parts) and S_M denotes the action of all matter fields (we use M to denote all matter fields, m to denote matter without pressure and r to denote radiation). In following derivations, we first take the variations of N, a, λ, L and then redefine Ndt as dt . We obtain

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} &= \frac{c}{3a^2L^2} + \frac{\lambda}{6a^4} + \frac{8\pi}{3}\rho_M, \\ \frac{2\ddot{a}a + \dot{a}^2 + \kappa}{a^2} &= \frac{c}{3a^2L^2} - \frac{\lambda}{6a^4} - 8\pi p_M, \end{aligned} \quad (2.3)$$

and

$$\begin{aligned} \dot{L} &= -\frac{1}{a}, \quad L = \int_t^\infty \frac{dt'}{a(t')} + L(\infty) \\ \dot{\lambda} &= -\frac{4ac}{L^3}, \quad \lambda = -\int_0^t dt' \frac{4a(t')c}{L^3(t')} + \lambda(0). \end{aligned} \quad (2.4)$$

We want to mention that we shall prove $L(\infty) = 0$ using the asymptotic solutions to be derived below. So, quite interestingly, aL is exactly the future event horizon. Besides, from eq.(2.3), it is easy to see that the $\lambda(0)$ term behaves the same way as radiation, thus can be a candidate for dark radiation [11]. For the purpose of solving equations of motion, we can always redefine $\lambda(t)$ as $\lambda(t) - \lambda(0)$, ρ_r as $\rho_r + \frac{\lambda(0)}{16\pi a^4}$, to let $\lambda(0) = 0$. From eq.(2.3), we can derive

$$\frac{\ddot{a}}{a} = -\frac{\lambda}{6a^4} - \frac{4\pi}{3}(\rho_M + 3p_M), \quad (2.5)$$

Note that we have $\lambda < 0$ for large enough time from eq.(2.4) ($c > 0$). We can ignore the matter effects for large enough time when dark energy is dominant. So eq.(2.5) implies the accelerating expansion of the universe.

Let us now try to find out solutions of the above equations. Fortunately, we can solve eqs.(2.3-2.5) exactly with general $c, \kappa, \rho_m \propto \frac{1}{a^3}$ and $\rho_r \propto \frac{1}{a^4}$. As a simple example, it is easy to observe that there is a de Sitter solution with ($a = e^{Ht}, L = e^{-Ht}/H, \lambda = -6H^2e^{-4Ht}$) and ($c = 6, \kappa = 0, \rho_M = p_M = 0$). We postpone discussing the general case to the next section.

Let us go on to discuss the asymptotic solutions for large enough time. Using eq.(2.3), we can derive

$$\frac{d(\dot{a}a)}{dt} = \frac{c}{3L^2} - \kappa + \frac{4\pi}{3}a^2(\rho_M - 3p_M). \quad (2.6)$$

Note that $\frac{c}{3L^2}$ is the only increasing function of time (L is a decreasing function) on the right hand of eq.(2.6), which becomes dominant for large enough time. So, we have

$$\frac{d(\dot{a}a)}{dt} = \frac{c}{3L^2} \quad (2.7)$$

asymptotically. Applying $\dot{L} = -\frac{1}{a}$, we can rewrite the above equation as

$$\frac{d^2a}{dL^2} = \frac{ca}{3L^2}, \quad (2.8)$$

with the general solution

$$a = c_1 L^{\frac{1-\sqrt{1+\frac{4}{3}c}}{2}} + c_2 L^{\frac{1+\sqrt{1+\frac{4}{3}c}}{2}} \sim L^{\frac{1-\sqrt{1+\frac{4}{3}c}}{2}}. \quad (2.9)$$

Using $\dot{L} = -\frac{1}{a}$ and the asymptotic solution $a \sim L^{\frac{1-\sqrt{1+\frac{4}{3}c}}{2}}$, we can derive

$$a \sim t^{\frac{1-\sqrt{1+\frac{4}{3}c}}{3-\sqrt{1+\frac{4}{3}c}}}, \quad c > 6 \quad (2.10)$$

$$a \sim e^{c_3 t}, c_3 > 0, \quad c = 6 \quad (2.11)$$

$$a \sim (c_4 - t)^{\frac{1-\sqrt{1+\frac{4}{3}c}}{3-\sqrt{1+\frac{4}{3}c}}}, \quad c < 6. \quad (2.12)$$

Eq.(2.12) implies that time would end at $t = c_4$. Using eq.(2.9) with $c < 6$, we can derive $aL \rightarrow 0$ when $L \rightarrow 0$ or equivalently $t \rightarrow c_4$. Thus the future event horizon shrinks to a point ($aL \rightarrow 0$) at $t = c_4$. Then time would end. In fact, below we shall see that we have $w < -1$ when $c < 6$. So there would be a big rip for the universe with $c < 6$.

From the state parameter of dark energy

$$w \equiv \frac{p_{de}}{\rho_{de}} = \frac{\lambda L^2 - 2ca^2}{3\lambda L^2 + 6ca^2} = \frac{-3 + 2c + \sqrt{9 + 12c}}{3(-3 - 2c + \sqrt{9 + 12c})}, \quad (2.13)$$

we get

$$\begin{aligned} w &= -1, & c &= 6, \\ -1 < w < -\frac{1}{3}, & c > 6, \\ w < -1, & c < 6. \end{aligned} \tag{2.14}$$

Note that

$$\rho_{de} \propto a^{-3(1+w)} > \rho_M, \tag{2.15}$$

consistent with our above assumption that dark energy is dominant for large enough time.

The asymptotic solutions eqs.(2.10,2.11) imply $a(\infty) \rightarrow \infty$ and eq.(2.12) implies $a(t = c_4) \rightarrow \infty$. So we have $L(\infty) \rightarrow 0$ or $L(t = c_4) \rightarrow 0$ from eq.(2.9). Thus, it is very interesting that aL is exactly the future event horizon. It should be mentioned that the above method can also apply to the initial HDE model. Using asymptotic solutions, we need not to assume that aL is exactly the future event horizon but only $\dot{L} = -\frac{1}{a}$. Then all the equations of the initial HDE model become local and depend on only the present initial conditions.

Let us comment on our results. First, it is the first model of holographic dark energy with action principle. Second, it implies the accelerating expansion of the universe. Third, eqs.(2.3,2.4) show that the evolution of the universe only depends on the present initial conditions a, \dot{a}, L, λ . So, it is clear that our HDE model obeys the law of causality. Furthermore, the use of the future event horizon as a cut-off is not an input in our model. Instead, the cut-off aL turns out to be the future event horizon automatically from equations of motion. So the long-standing problem of HDE “Why does the present evolution of the universe depend on the future of universe” has been solved. In fact, the evolution of the universe only depends on present conditions but equations of motion force the future event horizon to be the present cut-off aL magically. In a word, in this model we answer the above question by “It is natural that the future of the universe (future event horizon) depends on the present conditions (aL) of the universe.” Fourth, this model includes a de Sitter solution. Fifth, the energy density of dark energy

$$\rho_{de} = \frac{1}{8\pi} \left(\frac{c}{a^2 L^2} + \frac{\lambda}{2a^4} \right) \tag{2.16}$$

will remain positive if it is positive at the beginning and $\dot{a} > 0$. Finally, from eq.(2.2) it is interesting to note that $(-\lambda)$ is the conjugate momentum of the cut-off L . Or equivalently, λ is the energy associated with the conformal time $\eta = -L$.

2.2 The exact solutions

In this section, we shall solve equations eqs.(2.3-2.5) exactly with matter ($\rho_m = \frac{3b}{4\pi a^3}, p_m = 0$) and radiation ($\rho_r = \frac{3d}{8\pi a^4}, p_r = \frac{1}{3}\rho_r$). Thus, we have $\rho_M = \frac{3b}{4\pi a^3} + \frac{3d}{8\pi a^4}, p_M = \frac{d}{8\pi a^4}$. For simplicity, we focus on the case with $\kappa = 0$. It should be stressed that one can solve these equations exactly with general κ .

Redefine a new time $dL = -d\eta = -\frac{dt}{a}$, we can rewrite eq.(2.6) as

$$\frac{d^2a}{dL^2} = \frac{ca}{3L^2} + b. \quad (2.17)$$

Note that the above equation is independent of the parameter d . It seems that radiation does not affect the evolution of the scale factor a . This is however not the case. As we have mentioned before, we can always redefine λ as $(\lambda - \lambda(0))$, d as $(d + \frac{\lambda(0)}{6})$, to let $\lambda(0) = 0$ (note $\lambda(0)$ means $\lambda(t = 0)$ instead of $\lambda(L = 0)$). Then, using $\lambda(0) = 0$ we can relate the radiation parameter d to the scale factor a .

The general solution of eq.(2.17) is

$$a = c_1 \frac{1}{L} + c_2 L^2 + \frac{1}{3} b L^2 \ln L, \quad c = 6, \quad (2.18)$$

$$a = c_1 L^{\frac{1-\sqrt{1+\frac{4c}{3}}}{2}} + c_2 L^{\frac{1+\sqrt{1+\frac{4c}{3}}}{2}} - \frac{3b}{c-6} L^2, \quad c \neq 6. \quad (2.19)$$

Let us first discuss the case with $c = 6$. Using $dt = -adL$, we get

$$t = -c_1 \ln L - \frac{1}{3} c_2 L^3 - \frac{1}{9} b L^3 \ln L + \frac{b}{27} L^3. \quad (2.20)$$

From eq.(2.5), we can derive

$$\begin{aligned} \lambda &= -6[a^3\ddot{a} + ab + d] = -6[aa'' - a'^2 + ab + d] \\ &= -6d + \frac{2b^2L^2}{3} - 4bL^2c_2 + 12L^2c_2^2 - \frac{16bc_1}{L} - \frac{48c_1c_2}{L} - \frac{6c_1^2}{L^4} \\ &\quad - \frac{4}{3}b^2L^2 \ln L + 8bL^2c_2 \ln L - \frac{16bc_1 \ln L}{L} + \frac{4}{3}b^2L^2(\ln L)^2, \end{aligned} \quad (2.21)$$

with $a' = \frac{da}{dL}$. From the above equation, we can derive λ' while we can also derive $\lambda' = \frac{4a^2c}{L^3}$ from eq.(2.4). One can check that these two results are equal to each other, which implies that we have got the self-consistent solutions to eqs.(2.3-2.5). With eqs.(2.18,2.21), we can obtain the asymptotic state parameter as

$$\begin{aligned} w &\rightarrow -1, & L &\rightarrow 0, \\ w &\rightarrow -\frac{1}{3}, & L &\rightarrow L_0, \end{aligned} \quad (2.22)$$

where L_0 is defined by $a(L_0) = 0$, denoting the beginning time of the universe $t = 0$.

Now let us turn to discussing the case with $c \neq 6$ briefly. With eq.(2.19), we can derive

$$\lambda = -6[a^3\ddot{a} + ab + d] = -6[aa'' - a'^2 + ab + d] = -6d + \dots \quad (2.23)$$

One can check that eqs.(2.19,2.23) satisfy all of eqs.(2.3-2.5). From eq.(2.4), it is easy to observe that $\lambda(0) = \lim_{c \rightarrow 0} \lambda|_{c \rightarrow 0}$. Applying the condition

$$\lim_{c \rightarrow 0} \lambda|_{c \rightarrow 0} = -6(2bc_1 + d - c_2^2) = 0, \quad (2.24)$$

we can derive

$$c_2^2 = d + 2c_1b, \quad (2.25)$$

which shows that radiation does affect the evolution of the scale factor a eq.(2.19).

One can derive the asymptotic state parameter as

$$\begin{aligned} -1 < w &\rightarrow \frac{-3 + 2c + \sqrt{9 + 12c}}{3(-3 - 2c + \sqrt{9 + 12c})} < -\frac{1}{3}, & L \rightarrow 0, \\ w &\rightarrow -\frac{1}{3}, & L \rightarrow L_0, \end{aligned} \quad (2.26)$$

for $c > 6$ and similarly

$$\begin{aligned} w &\rightarrow \frac{-3 + 2c + \sqrt{9 + 12c}}{3(-3 - 2c + \sqrt{9 + 12c})} < -1, & L \rightarrow 0, \\ w &\rightarrow -\frac{1}{3}, & L \rightarrow L_0 \end{aligned} \quad (2.27)$$

for $0 < c < 6$.

2.3 Cosmology

In this section, we briefly discuss cosmology in our model. It is necessary to check that radiation is dominant in early time and $a \sim \sqrt{t}$ thus consistent with the standard cosmology.

In early time, we have

$$a(L_0) = 0, \quad a(L) \approx a'(L_0)(L - L_0), \quad a'(L_0) < 0. \quad (2.28)$$

Applying $\dot{L} = -\frac{1}{a}$, we get

$$t \approx -\frac{a'(L_0)}{2}(L - L_0)^2. \quad (2.29)$$

Thus, we obtain the expected result

$$a(t) \approx \sqrt{-2a'(L_0)t}, \quad (2.30)$$

implying that radiation is dominant in early time. Using the above solutions and $\lambda(0) = 0$, we can derive

$$w_{de} \approx -\frac{1}{3}, \quad \Omega_{de} \approx 0, \quad \Omega_r \approx 1, \quad (2.31)$$

which is consistent with eqs.(2.22,2.26,2.27) in the early time limit $L \rightarrow L_0$. Using the exact solutions eqs.(2.18,2.19), we can obtain the same results as above. So our model has passed the check that radiation is dominant in early time and $a \sim \sqrt{t}$. Thus, it would not ruin standard results such as nuclear genesis.

Applying the asymptotic or exact solutions, we can easily find that dark energy is dominant in the late time and state parameter behaves as

$$\begin{aligned} w &= -1, \quad c = 6, \\ -1 < w &< -\frac{1}{3}, \quad c > 6, \\ w &< -1, \quad c < 6. \end{aligned} \quad (2.32)$$

When $c = 6$, the universe will approach de Sitter space asymptotically. While for $c < 6$, dark energy will behave as phantom in late time and end up with a big rip. So it is necessary to estimate the value of c .

Since the universe has turned to the phase of accelerating expansion just recently, now we have $\frac{\ddot{a}}{a} \approx 0$ which leads to $\frac{\lambda}{6a^4} \approx -\frac{4\pi}{3}\rho_m$ from eq.(2.5). Using $aL \approx \frac{1}{H}$ together with $\rho_{de} \approx \frac{7}{3}\rho_m$, we have $\frac{c}{3a^2L^2} \approx \frac{cH^2}{3} \approx \frac{17}{20}H^2$. It follows that $c \approx 2.6$ which implies that, similar to the initial HDE model, dark energy will behave as phantom in late time. It is interesting to check our model with the observational data. We leave it to the future works.

Finally, we want to talk about the radiation-like term $\lambda(0)$ in our model. We can separate the energy density ($\frac{\lambda}{16\pi a^4} + \frac{c}{8\pi a^2 L^2}$) and pressure ($\frac{\lambda}{48\pi a^4} - \frac{c}{24\pi a^2 L^2}$) into eq.(2.3) into the dark energy part and dark radiation part:

$$\begin{aligned}\rho_{de} &= \frac{\lambda - \lambda(0)}{16\pi a^4} + \frac{c}{8\pi a^2 L^2}, \\ p_{de} &= \frac{\lambda - \lambda(0)}{48\pi a^4} - \frac{c}{24\pi a^2 L^2}, \\ \rho_{dr} &= \frac{\lambda(0)}{16\pi a^4}, \quad p_{dr} = \frac{1}{3}\rho_{dr}.\end{aligned}\tag{2.33}$$

By dark radiation, we mean that it has no interactions with other fields. It originates from the initial condition of λ and never decays. Interestingly, observational evidences support the existence of dark radiation [11]. We shall study this topic in details in the future work.

3 Models with other cutoff

In this section, we use the particle horizon and the Hubble horizon as the cut-off in the action. We find that, similar to the initial HDE model, neither of them can drive the accelerating expansion of the universe.

3.1 The particle horizon as the cutoff

In this subsection, we shall use the particle horizon as a cut-off. For simplicity, we focus on the case $\kappa = \rho_M = p_M = 0$. We find that this model has no accelerating solutions. Let us start with the action

$$S = \frac{1}{16\pi} \int dt [\sqrt{-g}(R - \frac{2c}{a^2(t)L^2(t)}) - \lambda(t)(\dot{L}(t) - \frac{N(t)}{a(t)})].\tag{3.1}$$

Following the same procedure of the above section, we obtain

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 &= \frac{c}{3a^2L^2} - \frac{\lambda}{6a^4}, \\ \frac{2\ddot{a}a + \dot{a}^2}{a^2} &= \frac{c}{3a^2L^2},\end{aligned}\tag{3.2}$$

and

$$\begin{aligned}\dot{L} &= \frac{1}{a}, \quad L = \int_0^t \frac{dt'}{a(t')} + L(0) \\ \dot{\lambda} &= -\frac{4ac}{L^3}, \quad \lambda = -\int_0^t dt' \frac{4a(t')c}{L^3(t')} + \lambda(0).\end{aligned}\tag{3.3}$$

From eq.(3.2), we can derive

$$\frac{\ddot{a}}{a} = \frac{\lambda}{6a^4}. \quad (3.4)$$

If $c > 0$, eq.(3.3) implies $\lambda < 0$ so that $\frac{\ddot{a}}{a} < 0$ for large enough time. On the other hand, eq.(3.2) implies $\lambda < 0$ if $c < 0$. So this model has no accelerating solutions in either case.

3.2 The Hubble horizon as the cutoff

Now let us turn to the case of the Hubble horizon as the cut-off. We focus on the case $\kappa = 0$. We begin with the following action

$$S = \frac{1}{16\pi} \int dt [\sqrt{-g}(R - \frac{2c}{L^2(t)}) - \lambda(t)(L(t) - \frac{N(t)}{H})] + S_M. \quad (3.5)$$

Following a similar procedure as the above sections, we obtain

$$(\frac{\dot{a}}{a})^2 = \frac{c}{3L^2} - \frac{\lambda}{6a^2\dot{a}} + \frac{8\pi}{3}\rho_M = -\frac{c}{3}(\frac{\dot{a}}{a})^2 + \frac{8\pi}{3}\rho_M, \quad (3.6)$$

$$\begin{aligned} \frac{2\ddot{a}a + \dot{a}^2}{a^2} &= \frac{c}{L^2} - \frac{\dot{\lambda}}{6a\dot{a}^2} - \frac{\lambda(\dot{a}^2 - \ddot{a}a)}{3a^2\dot{a}^3} - 8\pi p_M \\ &= -\frac{c}{3} \frac{2\ddot{a}a + \dot{a}^2}{a^2} - 8\pi p_M, \end{aligned} \quad (3.7)$$

and

$$L = \frac{a}{\dot{a}} = \frac{1}{H}, \quad \lambda = \frac{4a^3c}{L^3}, \quad (3.8)$$

From eqs.(3.6,3.7), we can derive

$$\frac{\ddot{a}}{a} = -\frac{c}{3} \frac{\ddot{a}}{a} - \frac{4\pi}{3}(\rho_M + 3p_M). \quad (3.9)$$

From eq.(3.6), it is clear that $(1 + \frac{c}{3}) > 0$. Then eq.(3.9) leads to $\frac{\ddot{a}}{a} < 0$. So there are no solutions of accelerating universe in this model.

4 Conclusions

In this paper, we propose a new HDE model with the action principle. This is the first time that one can derive a HDE model from an action. Furthermore, the causality problem has been solved completely in this and the initial HDE models. By introducing two fields λ and L , all equations of motion become local and the evolution of the universe is determined completely by present initial conditions. So clearly the HDE models obey the law of causality. Quite interestingly, the circular logic problem in HDE can also be solved. One always criticizes the original HDE model since the use of the future event horizon as a cut-off has assumed the accelerating expansion of the universe. Then how can one use an assumption based on the accelerating expansion to explain the accelerating expansion? In

fact, we do not need to assume the future event horizon as the cut-off but only a local equation $\dot{L} = -\frac{1}{a}$. Then the equations of motion magically force the cut-off to be exactly the future event horizon. This is also a support of the causality in HDE models. As we have shown, it is not that the present evolution of the universe depends on the future event horizon but that the future event horizon is determined by the present cut-off through equations of motion.

It is interesting to note that this new model is very similar to the initial one. For example, in both models, only the future event horizon rather than the particle or Hubble horizons can drive the accelerating expansion of the universe. Beside, both models have a pure de Sitter solution and behaves as phantom below the critical parameter($c = 6$ in the new model and $c^2 = 1$ in the initial one). The main difference between the two models is that the new one predicts the existence of dark radiation. We shall study this interesting problem in the following works.

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